Lecturered By Dr. H.K. Yadav.
Deptt. of Mathematics
Marwari College, Darbhanga
B.Sc - part-I (H) paper-I
Date - 17-04-2020

Set Theory

Q.1 Define a total order relation on a set and show by an example that a partially ordered set may fail to be totally ordered.

Soln. Total order relation: →

~~let (E, ≤) and (F, ≤) be~~

let $(E, \leq)$ is a totally ordered set and $(F, \leq)$ is a partially ordered set such that $E \cong F$, then $(F, \leq)$ is a totally ordered set.

A partially ordered set, even if it is totally ordered, need not have a least element. Then, the set R with natural ordering is tally ordered set but it does not have a least ~~number~~ element. One of the basic properties of the set N of natural numbers with the natural ordering is that N and every subset of N does not have a least element.

Q.2. Define addition of cardinal numbers and prove that $(\alpha+\beta)+\gamma = \alpha+(\beta+\gamma)$, where α, β, γ are any three cardinal numbers.

Soln: → Addition of cardinal numbers: → Let α and β be any two cardinal numbers and let X and Y be two disjoint sets with

$$\text{Card } X = \alpha, \quad \text{Card } Y = \beta$$

Then we define $\alpha + \beta = \text{Card}(X \cup Y)$

$$\Rightarrow \text{Card } X + \text{Card } Y = \text{Card}(X \cup Y).$$

Prove For any cardinal numbers α, β, γ.

We have to prove that $(\alpha+\beta)+\gamma = \alpha+(\beta+\gamma)$

proof: Let A, B and C be pairwise disjoint sets such that

$\alpha = \text{card}\, A,\ \beta = \text{card}\, B,\ \gamma = \text{card}\, C$

By associative law for union of sets

$\therefore (A\cup B)\cup C = A\cup(B\cup C)$

Now, $\alpha+\beta = \text{Card}(A\cup B),\ \beta+\gamma = \text{Card}(B\cup C)$

$\therefore (\alpha+\beta)+\gamma = \text{Card}\,(A\cup B)\cup C$

And $\alpha+(\beta+\gamma) = \text{Card}\, A\cup(B\cup C)$

Since, $(A\cup B)\cup C = A\cup(B\cup C)$

So, $(\alpha+\beta)+\gamma = \alpha+(\beta+\gamma)$

Thus the addition of cardinal numbers is associative.

Q.3. Define the concept of ordinal number and illustrate it by a suitable example.

Soln: Ordinal Number: → Let X be a well-ordered set then the order type of X is called the ordinal number of X.

Thus the ordinal number of a well-ordered set X is the order type of X.

Let $\{1,2,3,\dots m\}$ $(m\in N)$ and N have their natural orderings. Then these sets are well-ordered and hence totally ordered. We write

$\text{ord}\,\{1,2,3,\dots m\} = m,\ \text{ord}\, N = \omega.$

The ordinal numbers associated with the well-ordered sets,

$\phi\ \{1\},\ \{2\},\ \{1,2\},\ \{1,2,3\},\ \dots,\ \{1,2,\dots,m\},\dots$

are denoted by $0,1,2,3,\dots m$, respectively and each of these is called a finite ordinal number.

All other ordinal numbers are called transfinite ordinal numbers.